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Pilot-Wave Theory and Financial Option Pricing

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This paper tries to argue why pilot-wave theory could be of use in financial economics. We introduce the notion of information wave. We consider a stochastic guidance equation and part of the drift term of that equation makes reference to the phase of the wave. In order to embed information in financial option pricing we could use such a drift. We also briefly argue how we could embed information in the pricing kernel of the option price.

KEY WORDS: pilot-wave; Brownian motion; option pricing.

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1. INTRODUCTION

Asset prices such as stock prices are influenced by macroeconomic and psychological factors. Macroeconomic factors (see for instance Ross, 1976) have received ample attention in the economics and financial literature. On the issue of using psychological factors in explaining financial phenomena, we are in a more difficult situation. Important attempts have been made (Shiller, 2000) but the area is still welcoming new models.

In this paper we apply pilot-wave theory as developed by David Bohm (1952), in the theory of financial option pricing. A financial option is a contract where the buyer has the right to buy (or the right to sell) an underlying asset (such as a stock) at a certain price at a given date. An important question is to know what the price of such contracts should be. Option pricing theory has been developed by Black and Scholes (1973) and the third section of this paper provides for more details on this theory. In all of this theory, no mention is made of the role "information" may play in the price formation of such contracts (but see Chang and Chang, 1996, for some extensions). Indeed only macroeconomic factors are taken into account to explain the price trajectory. Such macroeconomic factors, can be explained in a physics environment, by a classical potential. However, psychological factors,

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which are also of importance in price formation, could be taken into account by a quantum potential representing an information potential. Although there is more often than not a strong connection between macroeconomic factors and psychological factors, this connection sometimes can be quite weak. This can be seen in the case of so called price bubbles where the prices of certain assets are in fact "hyped up" by massive fear (or on the contrary massive exuberance) amongst investors. The key ingredient in Bohmian mechanics is the pilot-wave and it is this wave which induces the quantum potential. Furthermore, the form (but not the intensity) of this wave is what counts. In Bohm and Hiley (1993) the pilot-wave is exemplified as a radio wave which steers a ship on automatic pilot. It is the form of that radio wave which counts, not its intensity. In Bohm and Hiley (1993) "information" is defined in a particular way. The notion of "active information," is information which has relevance to the movement of an electron itself and this notion of information has much less to do with, in the words of Bohm and Hiley (1993), "a quantitative measure of information that represents the way in which the state of a system is uncertain to us." The use of such "active information" has already been applied in other fields, notably by Khrennikov (2000) in his work on pilot-wave theory in cognitive-psychology models. We believe that this notion of "active information" is very much the type of information we need to better explain price behavior. We consider "active information" in our financial option pricing context as psychological information which can drive prices.

In this paper we focus on how pilot-wave theory could be used so as to embed information in financial option pricing. The paper is structured as follows. In the next section, we define why pilot-wave theory could be useful in financial economics. In the section following, we consider two brief applications of the pilot-wave theory in the financial theory of option pricing: (1) we consider the price of an option contract when information (of the pilot-wave) is embedded in the option contract; (2) we consider the price of the option when the pricing kernel is modified.

2. THE INFORMATION PILOT-WAVE: DEFINITION AND POSSIBLE CONNECTIONS WITH FINANCIAL ECONOMICS

Khrennikov (1999, 2003) provides for some good reasons why pilot-wave theory could be used in economics. The pilot-wave is seen as a wave of information. As in Khrennikov (2003), we have n traders and there is a configuration space, $\mathbf{S} = \mathbb{R}^n$ of prices, $\mathbf{s} = (s_1, s_2, \dots, s_n)$ where the price s_i is the price proposed by trader i. There is a price trajectory $\mathbf{s}_t = (s_1(t), s_2(t), \dots, s_n(t))$. From Bohmian mechanics (Bohm, 1952) we know then that

$$\frac{d\mathbf{s}_t}{dt} = \mathbf{v}^{\Psi}(\mathbf{s}_t), \tag{1}$$

where $\mathbf{v}^{\Psi} = (v_1^{\Psi}, v_2^{\Psi}, \ldots, v_n^{\Psi})$ is a velocity field on the configuration space $\mathbf{S}$. As in (Krennikov, 2003) we interpret the pilot-wave function in the economics/finance environment as a wave of information. The wave function does, as in the words of Khrennikov (2003) "describe the psychological influence of the price configuration $\mathbf{s} = (s_1, s_2, \ldots, s_n)$ to the traders." This wave of information guides the price.

If we want to consider a financial version of the Schrödinger equation then we must (1) argue for the use of some financial "equivalent" of the Planck constant and, (2) provide for a financial interpretation of the potential function. Both issues are not evident. In regard to the Planck constant, Khrennikov likens the Planck constant to a price scaling (possibly time dependent) parameter in a financial environment. Would it be a rate of interest? In some sense such rate of interest could have a relationship to an inherent level of uncertainty in the economy which is not eliminatable. We leave the "debate" open and will denote the "financial" equivalence of $\hbar$ as $e_{\$}$.

In regard to the potential function, Khrennikov (2003) advances that the potential function in an economic context, would measure external economic conditions and potential interactions between the n traders. For instance, we could opt for a free particle potential function when the economy has an almost limitless capacity. Or to specifically capture interaction between traders, we could see the potential function as the summation of the quadratic difference of prices held by a each couple of traders. Related to this we could consider the potential function to be a constant function equal to the risk free rate of interest (i.e. the rate of return on a nondefaultable government bond).

For a given economic potential function, the information–wave function could be supposed to evolve as the *financial* Schrödinger equation:

$$\widehat{H}\Psi(\mathbf{s}, t) = i e_{\$} \frac{\partial \Psi(\mathbf{s}, t)}{\partial t}, \tag{2}$$

where $\Psi : \mathbf{S} \times \mathbb{R} \to \mathbb{C}$ and is square integrable. The Hamiltonian operator is here interpreted as in (Krennikov, 2003) as the operator corresponding to the financial energy given by the function $H(\mathbf{s}, \mathbf{p})$, where $\mathbf{p}$ is price momentum. We remark that the fundamental assumption in the "Khrennikov-approach" is thus that $\mathbf{s}_t$ (and hence $\mathbf{v}^{\Psi}(\mathbf{s}_t)$) can be derived by assuming there exist a function of two variables $\mathbf{s}$ and t that satisfies the Schrödinger equation.

Central to Bohmian mechanics, is that the wave function defines the quantum potential. We can define the information-value wave function as

$$\Psi(\mathbf{s}, t) = R(\mathbf{s}, t) \exp(i S(\mathbf{s}, t)), \tag{3}$$

where $R(.,.)$ and $S(.,.)$ are respectively the amplitude and phase of the wave function.

Then we can define, as in Khrennikov, the financial information potential (quantum potential) as

$$U(\mathbf{s}, t) = -\frac{1}{R} \sum_{i=1}^{n} \frac{\partial^2 R}{\partial s_i^2}. \tag{4}$$

The information "force" is then

$$-\frac{\partial U}{\partial s_i}(\mathbf{s}, t). \tag{5}$$

Thus, by using the Schrödinger equation (assuming that $\Psi(\mathbf{s}, 0)$ is known) we know the value of $\Psi(\mathbf{s}, t)$ and hence we can calculate the quantum potential then at time t as in the above equation.

3. PILOT-WAVE THEORY IN OPTION PRICING: PRICE AND PRICING KERNEL

The modelization of a stock price process is performed with the help of a geometric Brownian motion. Such a Brownian motion is central in the derivation of a so called option price. Option prices are basically prices of contracts which stipulate the right to buy or sell an asset at a certain price in the future. The solutions to the celebrated Black–Scholes PDE, for which Scholes and Black (1973) won the Nobel prize, provide for the theoretical prices of option contracts. The Black–Scholes PDE is in effect a Kolmogorov Backward equation. We want to discuss two issues in this last section of the paper:

1. the price of an option contract when information (of the pilot-wave) is embedded in the option contract
2. the price of an option contract when the pricing kernel is modified

Bacciagaluppi (1999) introduces a stochastic guidance equation (but see also Bohm and Hiley, 1993) which in Itô form consists of two main elements a drift coefficient and a diffusion coefficient. It is defined as follows:

$$d\mathbf{s} = \left(\frac{\hbar}{m} \nabla S + \alpha \frac{\hbar}{2m} \frac{\nabla |\psi|^2}{|\psi|^2} \right) dt + \sqrt{\alpha}\, d\omega, \tag{6}$$

where ∇S is the gradient of the phase of the wave function, α is a parameter and $d\omega$ is a Wiener process with the constraints that $\overline{d\omega} = 0$ and $\overline{(d\omega)^2} = \frac{\hbar}{m}$. If $\alpha = 1$, then we obtain Nelsonian mechanics (Nelson, 1966) and we do not expand on this type of mechanics in this paper. For the purposes of this paper we are interested in $\alpha = 0$, which corresponds to Bohm's theory. We could may be give an economic interpretation to the phase of the wave function as some indicator of how traders interact on the level of information. It is important that we come up with a much finer interpretation of what the financial analogue could be of the phase. This will be the object of further research. What we can say for now, is that if one takes the part $\frac{1}{m}\nabla S$ out of the above stochastic guidance equation we can interpret that part as the average velocity of the particle (Bohm and Hiley, 1993). In financial terms, this velocity would need to contain the growth rate by which the stock price (the particle) grows over time.

In option pricing theory we use a geometric Brownian motion:

$$d\mathbf{s} = \mu \mathbf{s}\, dt + \sigma \mathbf{s}\, d\omega^*, \tag{7}$$

where $\overline{d\omega^*} = 0$ and $\overline{(d\omega^*)^2} = 1$ and σ is stock price volatility (i.e. the square root of the stock price variance). If we denote the transition probability density $p(\mathbf{s}, t; \mathbf{s}', t')$ which follows the above geometric Brownian motion, then p will also satisfy the backward Kolmogorov equation:

$$\frac{\partial p}{\partial t} + \frac{1}{2}\sigma^2 \mathbf{s}^2 \frac{\partial^2 p}{\partial \mathbf{s}^2} + \mu \mathbf{s} \frac{\partial p}{\partial \mathbf{s}} = 0. \tag{8}$$

It can be shown that this equation is close to the so called Black–Scholes PDE which yields option prices. Amongst the differences the above PDE has with the Black–Scholes PDE there is one important difference: $\mu = r_f$, where r_f is the risk free interest rate. Our proposal is then as follows: if we want to incorporate information (as derived from the pilot-wave) then we could consider a geometric Brownian motion (like Eq. (7)) where instead the drift is now $\mu = e_\$ \nabla S + r_f$. We leave in this paper our interpretation of the financial equivalent of $\hbar$ (denoted by $e_\$$) open. As was the case with the financial interpretation of the phase, here also the interpretation of a financial analogue of $\hbar$ will be the object of further research.

In that sense, we obtain an option valuation which now would embed information and the so called risk neutral property of the Black–Scholes PDE (i.e. the PDE does not use the risk free rate of interest) may then not be maintained.

We can also consider another way of pricing options: the way of the pricing kernel. In words, one can write the option price by using the Feynman–Kac formula. The pricing kernel is nothing else than a conditional probability which lets the payoff function evolve backwards in time so the option price can be found. Intuitively, what will the pricing kernel be if we were to *only* consider information? In our pilot-wave theory proposal, we would get a Feynman path integral containing the Planck constant. So a possible topic for further research is to consider what type of pricing kernel we would have when information is embedded in pricing.

Finally, we could may be argue that by embedding information (out of the pilot-wave) we can make a case for arbitrage. Arbitrage can probably be best defined as the realization of an abnormal return on an investment without incurring any risk. In efficient markets, information will decay fast. Hence, arbitrage opportunities will be quickly eliminated in such markets. However, one might believe that arbitrage opportunities might linger because in reality markets are not as efficient as theory predicts, so information does not decay as quickly as predicted. In this case we may want to alter the non-arbitrage based pricing model. We could write μ in the drift of Eq. (7) as $\mu = e_{\$}\nabla S + r_f + \xi$, where ξ represents some "chaotic" contribution to the velocity of the stock price. This would be following the spirit of Bohm and Vigier (1993). This chaotic contribution could be assumed to be the result of arbitrage. In that sense, we would find the Black–Scholes option price as a special case of a much more general Black–Scholes type PDE.

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